

# **Interactive Learning of Geometry by Using Technology: Examples and Steps to its Implementation in the Classroom**

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**Abstract:** Dynamic geometry tools may support and enrich a more constructive learning process with many activities by the students. Using or integrating pedagogically conceived WWW-resources or webbased learning environments with Java-tools are further steps to "new ways of learning" with more student activities, collaborative working and interactive tools for exploring and discovering math. These tools are more theoretically known by many German teachers. One has to face first lots of problems and purposes that it makes it difficult to integrate new technologies as a matter of course in the classroom. The following examples will show some efforts to spread the idea of using technology in geometry classes for the teacher's need e.g. how to get firm with new technologies with the help of in-service teacher training or by pedagogically designed websites on the internet. On the other hand I would like to show some HTML- or Java-based material drafted for the use in the classroom for individual and self-responsible exploration.

## **1. *learn:line*, the Educational Server: Spreading the Idea of Individual Learning (of Geometry Supported by Technology and Interactive Tools)**

### **A Platform for Information, Communication and Cooperation**

Learning in our information society means to face many different specialized, inter-disciplinary or social problems. So school has to take care of more than teaching factual knowledge and special information of each school subject. Teaching aims at lifelong learning as a perspective of being prepared in this changing world. Students have to be encouraged and taught to continue developing their knowledge network. By including the new possibilities of new media- especially telecommunication- we hope to improve the quality of working and learning in school and further education. We would like to support a more responsible, constructive and communicative learning in open learning situations where the learning individual could better contribute information from his own experiences and actual interests in common work. We would like to initiate learning with exploration, communication and cooperation.

The educational server *learn:line* (<http://www.learn-line.nrw.de>) is a platform on the internet for information, communication and cooperation. We want to support possibilities, to get in contact with other people, to exchange experiences, to work together in projects and to learn from each other. The latest academic research of learning processes leads us to create learning situations that are not only linear structured but open, explorative, situations, in which learners have to play an active role. This will change the teacher's role, too- from an "instructor" to a "learner enabler", someone who advises and accompanies the individual learning process.

New media may play an important role. One may not forget that media competence is an assumption for acting in a socially responsible way. The extension of media competence is one of the most important aims. By working and using new and traditional media students will learn to use them in a creative and appropriate way, knowing their chances and limits.

## Work Areas –Characteristics of the NRW-Educational Server

Work areas relating to particular topics are characteristic of the NRW-Educational Server. They represent a kind of learning arrangement or provide an infra-structure for encouraging learning. Each work area that concentrates on a particular topic exists as an independent information and communication offer in its own right; it provides the relevant material while still encouraging the user to some feedback. Each work area is divided into the following sections: a *media centre*, a *foyer* and a *noticeboard*.

In the *media centre* the user will find information, selected materials and suggestions. Links to other sites will contribute to work on the topic and to build up a „didactic community“. In the *foyer* interested parties can get the chance to present their own work related to the topic, results and experiences from a particular project.

The *noticeboard* is made available in every work area for questions, answers, suggestions and discussion.

The *work room* is a virtual place, that student can easily access with a browser where they can work together on documents, exchange ideas or files of their work.

## Computational Geometry - a Work Area Especially for The Teacher's Need

From the learn:line homepage by choosing *subjects* -> *math* interested parties can gain access to the work area „computational geometry“. They will find opportunities to use the internet to accompany in-service teacher training concerning computational geometry for k12-teachers. An "online tutorial" demonstrates the special "educational power" – focussed on teachers who only have little knowledge and experience in dynamic geometry with programs like Cabri, Sketchpad, or the German Euklid (most of the German teachers did not learn about it in pre-service teacher training). They can see the possibilities and chances when using these tools in the classroom with the help of examples and a lot of constructed figures. In addition to this teachers may find short descriptions about these good new media, evaluated "exemplary" for a more constructive way of learning. Reports comparing similiar products and pointing out main differences are as well available as ideas for its use in classroom, possibilities to exchange Cabri- or Sketchpad-Files and pointers to other websites that use or discuss these math tools.

The *foyer* of this work area shows reports of classroom situations, results and classroom materials that had been sent by other teachers and could be discussed and improved.

<http://www.learn-line.nrw.de/Faecher/Mathematik/Geometrie/medfoy/medioe.htm>



figure 1: media centre of the work area

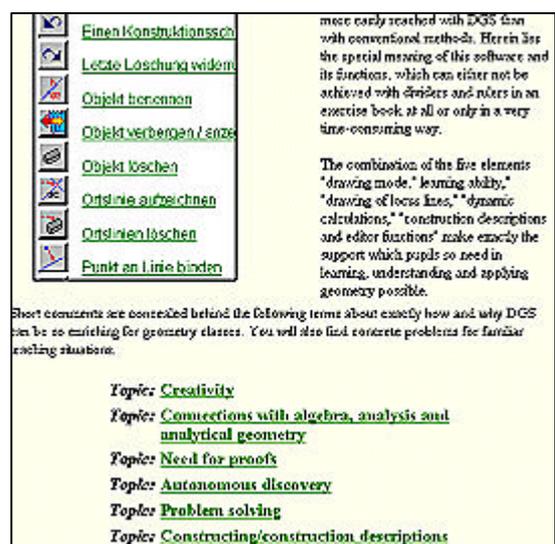


figure 2: page of the "online tutorial"

## 2. Materials, Exercises, Special Interactive Geometry Projects - Especially for the Classroom

### New Ways of Learning and Teaching

There are many problems associated with the nature of teaching math, especially geometry:

- visualization of geometric properties in the range of its validity,
- teaching, how to proof theorems,
- showing the connection between geometric theorems,
- lack of simple real life problems that lead to a geometrical problem.

Today the academic research reminds us of the learning process being first of all an individual, autonomous, exploratory and self-responsible process, which will then be highly efficient if it takes place in a rich environment.

A change from the focus on the teacher and the teaching process to the learner and the learning process will demand a change of the teacher's role. They will have to change from being an instructor to a 'learner enabler', somebody who accompanies and supports the individual learning process. The learners will have to change from being a mere recipient to an active creator of their learning process. They have to be encouraged and enabled to take control of their own learning process, which means defining the aim, selecting the content and evaluating what they have learned.

In this classroom situation both teachers and learners need appropriate materials and tools to support the learning process mentioned above.

Here dynamic, interactive programs and hypertexts, too, play an important role since the first generation of dynamic tools has been created. Since the internet and java-capable browsers have become more popular and can be used in school because of better hardware in the computer-labs teachers have more possibilities for getting information, for communication, interactivity, cooperation, for exchanging materials ready for use, etc.

### Example: Ka's Geometriepage und Mathe-Galerie

My private website is meant to be additional to the work area of the educational server learn:line. There I'd like to support teachers in experimenting with interactive dynamic geometry tools. Like the famous *math forum in Swarthmore* I would like to offer well-working classroom materials, lesson plans and ideas, that have often been discussed and modified in teacher training. German teachers start to use the corresponding discussion area more tentatively than their colleagues, for example in the United States- perhaps because of the lack of connection with the WWW at home and the costs for telecommunication that are still rather high in Germany.

In a special part of my website, called projects, I'm going to present classroom situations, where dynamic tools will gain ground for discovering and pursuing individual ways of learning and broaden the network of cognitive concepts. Here is one of my projects:

Supposing that learning takes place in an individual way and knowing that different students will choose different explanations and examples to understand essential geometric theorems and their proofs, I drafted a learning unit based on a worksheet about the Pythagorean theorem and its different proofs. I collected different proofs with a wide range of interactivity, knowing that students prefer different accesses to these proofs and related theorems. Some of the downloaded resources only visualize the theorem or are hypertexts that make understand the idea of the proof before it is written down in a more formal language. Some of the chosen websites contain java applets allowing the students to manipulate and experimentate interactively with dynamic constructions.

In addition to this, students could use the PC-versions of Cabri, Sketchpad or Euklid to prepare some explanations for the other students.

I used this learning unit in a ninth grade geometry class to stimulate the students to come up with their own way of building up their knowledge network; this meant that pupils have to decide for themselves what information is important, how much and what sort of help they require. After a period of individual work the students who worked in groups of two or three had to present their investigations and their results to all students of the class. It seems to be evident that discussion in a group of two or three could intensify the real understanding of a given context. In addition to this better students had to analyse informations at a higher level.

<http://www.ham.nw.schule.de/projekte/swmathe/Uonline/>



figure 3: geometry projects of Ka's Geometriepage und Mathe-Galerie

### 3. Integrating Webbased Materials and Java-Resources in Geometry Courses

#### Using (Creating ) Interactive Dynamic Geometry (Java-) Resources

Besides the PC-version of dynamic geometry tools like Cabri, Sketchpad or Euklid Java-based constructions sometimes can complete demonstrations and geometric constructions for exploration in the classroom.

Some of the special advantages of these Java-applets are:

- complex constructions are just ready for use and can be combined with worksheets, that are appropriate for the special classroom situations
- most java applications for geometry constructions only "allow" manipulations that are provided by the constructor himself (an advantage for special problems where students otherwise can "destroy" the construction)
- the applicatons run at any platform and do not require a special program
- the teacher can build up a library ready to use and integrate in his own concepts or in the intranet.

Here is an example in which way a geometry applet may be integrated and modified for a special need: The applet comes from "Manupula Math", a Japanese website that offers more than 100 tiny well-working applets. It runs as well in the intranet as in the internet.

<http://kunden.swamm.de/Geometriepage/mseties.htm>

Some other could be found:

*Cabri Java*

<http://www-cabri.imag.fr/projets/cabrijava.html>

*JavaScetchpad:*

[http://www.keypress.com/sketchpad/java\\_gsp/](http://www.keypress.com/sketchpad/java_gsp/)

*Cut-the-knot*

<http://www.cut-the-knot.com/>

*IcosaWeb*

<http://www.guetali.fr/home/berdel/maths/cours/geom.htm>

## 4. Learning Environments and How it May Work

### In General

Good learning-environments based on hypermedia are advantageous for learning and understanding mathematics. The understanding of math requires the knowledge of details, conclusions and relations between single objects. If subjects are offered in a linked-up, but not linear structure it will be easier to build up a network of mathematical, geometric knowledge in the learner's mind.

That's why good learning environment should offer some important "details", e.g.:

- guided tours
- survey of the learning subject, table of contents, glossary
- meta level map (where am I?)
- some (recommended) provided paths
- different modes of representation and levels of activity
- hypertextual structures with internal and external relations
- some helpful biographical facts of famous mathematicians, real-life application,...
- exercises and contextual help
- possibilities to integrate other documents and visualizations- either static or dynamic, interactive tools and demonstrations

A glance at the market of new offline-media shows that only a few products match up to almost all criteria so that they could be called "exemplary" in the way mentioned at the beginning. Learning environments in the WWW have even more advantages:

- HTML-based hypertexts are open,
- can easily be modified and being published in the internet,
- they can be improved, developed, discussed
- can include practical advice for teachers, etc.

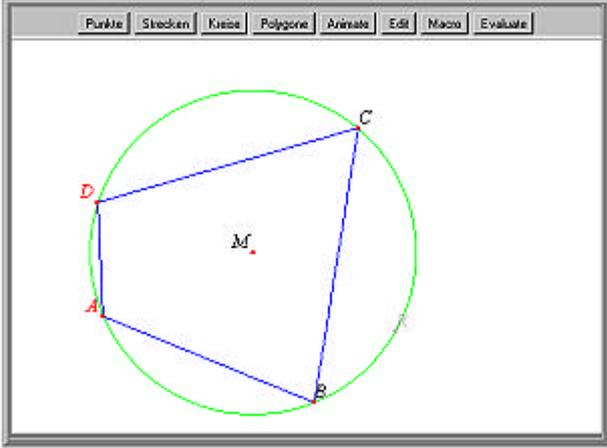
"Exemplary" learning-environments for geometry (written in German) do not exist at the moment. But we will find some hopeful steps to using technology for individual, meaningful, exploring, that means, constructive learning:

### Computational Geometry with the Java-based GEONET

At the math department of the University of Bayreuth (Germany) the Java-based dynamic geometry tool **GEONET** has been developed. Based on this tool there are some little hypertexts that offer different ways through the subject, for example "the order of the quadrilaterals" and some (applied) exercises, too. Students of math education have dealt with further subjects (Thales, Pythagorean theorem).

Im folgenden soll die experimentelle Aussage durch einen mathematischen Beweis bestätigt werden. Wir wollen den Beweis nicht einfach vorführen - es soll vielmehr durch die gestellten Fragen möglich sein, ihn selbst zu erarbeiten.

**Satz: Im Sehnenviereck beträgt die Summe gegenüberliegender Winkel jeweils  $180^\circ$**



• [Verbinde die Ecken](#) des Sehnenvierecks mit dem Mittelpunkt M. Welche gemeinsame Eigenschaft haben die Teildreiecke MAB, MBC, MCD, MDA?

• Bezeichne folgendermaßen: Winkel MAB = alpha, Winkel MBC = beta, Winkel MCD = gamma, Winkel MDA = delta.

• Berechne (unter Ausnutzung der oben erwähnten Eigenschaft) die Summe der Winkel DAB + BCD und AEC + CDA abhängig von alpha, beta, gamma und delta.

• Wie groß sind die eben ausgedrückten Winkelsummen, wenn man die Winkelsumme im Viereck mitberücksichtigt?

• Um den Beweis zu vervollständigen müssen noch [zwei weitere Fälle](#) behandelt werden! Überlege, ob man analog schließen kann.

Figure 4: learning environment including the java-based dynamic geometry program GEONET

## **The Pilot Scheme "self-learning environments for college math"**

At the beginning of this month a big pilote scheme has started in Germany. North Rhine Westfalia takes place with math education. Its aim is to analyze and draft examples how arrangements and situations of learning mathematics in the college must be organized so that students may temporarily work in groups without a teacher (maybe with the help of a „teacher on demand“ by using the internet). 15 teachers of 5 schools work on this projects, supported by publishers of multimedia or schoolbooks. One year later 25 other selected schools and teachers will test and evaluate the drafted material.

We are sure that these periodes where students have to organize their tasks by themselves and work at their own have to be supported by software like geometry tools, CAS and other interactive programs and ressources of the internet. We would like to invite everyone to take part of this project with advice, materials, good ideas or suitable software. You could find out more details on our website (<http://www.learn-line.nrw.de/Faecher/Mathematik/MV/>) and may contact our project-coordinator by email ([guido.von-saint-george@mail.lsw.nrw.de](mailto:guido.von-saint-george@mail.lsw.nrw.de)).

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